Magneto-electric Effect and Magnetic Charge

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Abstract

It is shown that both the electric and magnetic dipole moment vectors of hydrogen atom in the excited states with wave function

$$u_n^{(\pm)} = \frac{1}{\sqrt{2}}[R_{n,n-1}(r)Y_{n-1,\pm(n-2)}(\theta\varphi) \pm R_{n,n-2}(r)Y_{n-2,\pm(n-2)}(\theta\varphi)]$$

align themselves in the direction of an external uniform electric field which is characteristic of magneto-electric effect. These states are found to have magnetic charge $g = \frac{3n}{(n-2)e}$ on account of this effect. This result is confirmed by an independent method. An experiment is suggested to fabricate these states and detect the magnetic charge. It may be worth noting that inspite of many experimental searchs, magnetic charge, whose existence has been theorized both in electrodynamics and non-abelian gauge theories, none have been found so far, nor there exist any suggstion as to where these are to be found.

Dirac [1] incorporated magnetic charge (monopole) in electrodynamics by introducing a string singularity in the vector potential and showed that its strength is given by the relation

$$g = \frac{n}{2e}, \quad n = 1, 2, 3... \tag{1}$$

This relation was independently obtained by Saha [2] by quantizing the angular momentum of a two-body system consisting of a magnetic point charge and an electric point charge. Existence of magnetic charge is also a feature of certain non-abelian gauge theories [3]. although numerous experimental searches [4] have been undertaken to detect them, none have been found so far, nor any suggestion has been made as to where to find them. In this communication we show that it is possible to fabricate magnetically charged states by superposition of excited states of hydrogen atom by placing it in a uniform electrostatic field and detect their magnetic charge.

We have shown in an earlier publication that when an electrically charged particle is placed in a magneto-electric medium it acquires a magnetic charge [5] . A magneto-electric medium exhibits magnetic (electric) polarization when placed in an external electric (magnetic) field [6]. An assembly of particles possesing both permanent electric and magnetic dipole moments is an example of such a medium. When placed in an electric field the electric dipoles align in the direction of this field producing electric polarization. This also produces magnetic polarization because the magentic dipoles get aligned.

It transpires that certain superposition of excited states of hydrogen atom posses both permanent electric and magnetic dipole moments. The states with wavefunctions

$$u_n^{(\pm)} = \frac{1}{\sqrt{2}}[R_{n,(n-1)}(r)Y^{(\theta\varphi)}_{(n-1),\pm(n-2)} \pm R_{n,(n-2)}(r), Y^{(\theta\varphi)}_{n-2,\pm(n-2)}] \tag{2}$$

are formed when the atom is placed in an external uniform electric field. The electric

and magnetic moments along the external field direction are, found to be [7]

$$d^{(e)} = \mp \frac{3n}{2e} \tag{3}$$

$$d^{(m)} = \pm \frac{(n-2)e}{2} \tag{4}$$

which demonstrates that the electric field produces both electric and magnetic polarization.

The generic representation of the magneto-electric polarization is

$$\begin{aligned} \vec{P}_e &= \chi_e \vec{E} + \chi_{me} \vec{B} \\ \vec{P}_m &= \chi_m \vec{B} + \chi_{me} \vec{E} \end{aligned} \tag{5}$$

where $\vec{P}_e$ and $\vec{P}_m$ are electric and magnetic polarization vectors, χ_e and χ_m are electric and magnetic susceptibilities and χ_{me} is the magneto-electric susceptibility. In terms of the dielectric constant $\in = 1 + \chi_e$ and magnetic permeability $\mu = \frac{1}{1-\chi_m}$, the above relations take the form

$$\begin{aligned} \vec{D} &= \vec{E} + \vec{P}_e = \in \vec{E} + \chi_{me} \vec{B} \\ \vec{H} &= \vec{B} - \vec{P}_m = \frac{1}{\mu} \vec{B} - \chi_{me} \vec{E} \end{aligned} \tag{6}$$

For the atomic case there is no medium, so that

$$\in \mu = 1 \tag{7}$$

and relations (6) can be written as

$$\begin{aligned} \vec{D} &= \in \vec{E} + \chi_{me} \vec{B} \\ \vec{H} &= \in \vec{B} - \chi_{me} \vec{E} \end{aligned} \tag{8}$$

For the case when $\vec{B} = 0$

$$\begin{aligned} \vec{D} &= \in \vec{E} \\ \vec{H} &= -\chi_{me} \vec{E} \end{aligned} \tag{9}$$

In this equation if we take $\vec{E}$ as the electric field generated by the electron, a magnetic field $\vec{H}$ is produced through the magneto-electric effect whose divergence gives the magnetic charge density

$$\rho^{(m)} = -\vec{\nabla} \cdot \vec{H} = \chi_{me} \vec{\nabla} \cdot \vec{E} = \frac{\chi_{me}}{\in} \vec{\nabla} \cdot \vec{D} = \frac{\chi_{me}}{\in} \rho^{(e)} \tag{10}$$

This gives us the magnetic charge

$$g = \int d^3 x \rho^{(m)}(x) = \frac{\chi_{me}}{\in} \int d^3 x \rho^{(e)}(x) = e(\frac{\chi_{me}}{\in}) \tag{11}$$

a relation derived by the author in an earlier publication [5].

For the atomic states under consideration the energy shift ΔE in an external field $\mathcal{E}$, along the z-axis are

$$\nabla E^{(\pm)} = \mp \frac{3n}{2e} \mathcal{E} \tag{12}$$

Since, on account of (8), the applied electric field $\mathcal{E}$ is equivalent to a magnetic field $\mathcal{B}$ given by

$$\mathcal{B} \equiv \frac{\chi_{me}}{\in} \mathcal{E} \tag{13}$$

the same energy shift can be expressed as

$$\nabla E^{(\pm)} = \pm \frac{(n-2)e}{2} \mathcal{B} = \pm \frac{(n-2)e \chi_{me} \mathcal{E}}{\in \mathcal{E}} \tag{14}$$

Equating (12) and (14) and making use of (11) we get

$$eg = +(\frac{3n}{n-2}) \tag{15}$$

for the electron wich has negative charge. This Dirac-Saha type relation is valid for $n > 2$ since for n = 2, the magnetic polarization produced by the electric field vanishes; there is no magneto-electric effect.

The formula (15) derived above can be obtained by an independent method which is outlined below. The basic relation used in this method is that the two definitions of electric dipole moment $\vec{d}_e$

$$\vec{d}_e = \int d^3r\rho^{(e)}(\vec{r}) \tag{16}$$

and

$$\vec{d}_e = \frac{1}{2}\int d^3r \times \vec{J}^{(m)}(\vec{r}) \tag{17}$$

where for the static case

$$\vec{J}^{(m)} = \vec{\nabla} \times \vec{D} \tag{18}$$

$$\rho^{(e)} = \vec{\nabla} \cdot \vec{D} \tag{19}$$

are identical. Eqn (17) tells that a closed magnetic current is equivalent to an electric dipole just as closed electric current is equivalent to a magnetic dipole. The equality between (16) and (17) can be proved by making use of (18) and (19) which gives

$$\int (d^3r) r_i \rho^{(e)}(\vec{r}) = -\int (d^3r) D_i + \int (d^3r)\partial_j (r_i D_j) \tag{20}$$

$$\int (d^3r)[\vec{r} \times \vec{J}^{(m)}(\vec{r})]_i = -\int (d^3r) D_i + \frac{1}{2}\int (d^3r)[\partial_j(r_i D_j) - \partial_i(r_j D_j)] \tag{21}$$

The second terms in eqns (20) and (21) are integrals over surface which can be put at infinity where the fields vanish on account of vanishing of hydrogen atom wavefunctions. We thus have

$$\int (d^3r)\vec{r}\rho^{(e)}(\vec{r}) = \frac{1}{2}\int (d^3r)[\vec{r} \times \vec{J}^{(m)}(\vec{r})] \tag{22}$$

In order to obtain any result from this identity we have to have expressions for $\rho^{(e)}$ and $\vec{J}^{(m)}$. It is convenient to write these in terms of second quantized field operators $\psi_n^{(\pm)}(\vec{r})$: for the states with wavefunctions $u_n^{(\pm)}$

$$
\begin{aligned}
\rho^{(e)} &= e(\psi_n^{(+)^+}\psi_n^{(+)} + \psi_n^{(-)^+}\psi_n^{(-)}) = e(\rho^{(+)} + \rho^{(-)}) \\
\rho^{(m)} &= g(\psi_n^{(+)^+}\psi_n^{(+)} - \psi_n^{(-)^+}\psi_n^{(-)}) = g(\rho^{(+)} - \rho^{(-)}) \\
\vec{J}^{(e)} &= \frac{ie}{2}(\psi_n^{(+)^+}\overline{\nabla}\psi_n^{(+)} + \psi_n^{(-)^+}\overline{\nabla}\psi_n^{(-)}) = e(\vec{J}^{(+)} + \vec{J}^{(-)}) \\
\vec{J}^{(m)} &= \frac{ig}{2}(\psi_n^{(+)^+}\overline{\nabla}\psi_n^{(+)} - \psi_n^{(-)^+}\overline{\nabla}\psi_n^{(-)}) = g(\vec{J}^{(+)} - \vec{J}^{(-)})
\end{aligned}
\tag{23}
$$

where $a\overline{\nabla}b = a(\vec{\nabla}b) - (\vec{\nabla}a)b^2$. These are so constructed as to satisfy the parity transformation properties

$$
\begin{aligned}
P\rho^{(e)}P^{-1} = \rho^{(e)}, &\quad P\rho^{(m)}P^{-1} = -\rho^{(m)} \\
P\vec{J}^{(e)}P^{-1} = -\vec{J}^{(e)}, &\quad P\vec{J}^{(m)}P^{-1} = +\vec{J}^{(m)}
\end{aligned}
\tag{24}
$$

It can be verified, using the parity properties of the wavefunctions $u_n^{(\pm)}$ that

$$
\begin{aligned}
P\rho^{(+)}P^{-1} = \rho^{(-)}, &\quad P\rho^{(-)}P^{-1} = \rho^{(+)} \\
P\vec{J}^{(+)}P^{-1} = -\vec{J}^{(-)}, &\quad P\vec{J}^{(-)}P^{-1} = -\vec{J}^{(+)}
\end{aligned}
\tag{25}
$$

This ensures that (24) is satisfied. Using (23) in the lefthand side of (22) for the z-component, and substituting (16)

$$
< n^{(\pm)} \mid d_3^{(e)} \mid n^{(\pm)} >= \mp\frac{3n}{2e} \tag{26}
$$

whereas substitution of the right hand-side in (17) gives

$$
< n^{(\pm)} \mid d_3^{(e)} \mid n^{(\pm)} >= \mp\frac{(n-2)g}{2} \tag{27}
$$

$< n^{(\pm)} >$ being the state with wavefunction $u_n^{(\pm)}$. Equating (25) and (26) gives,

$$
eg = \frac{3n}{n-2} \tag{28}
$$

as obtained by consideration of magneto-electric effect.

In order to experimentally detect the magnetic charge, it is best to take the state with n=3 since it is easiest to excite which can be done with appropriate laser beams. A

beam of such excited atoms is then passed through a uniform electric field which among other Stark states prepares our desired states

$$
\begin{aligned}
u_3^{(+)} &= \frac{1}{\sqrt{2}}(R_{32}Y_{21} + R_{31}Y_{11}) \\
u_3^{(-)} &= \frac{1}{\sqrt{2}}(R_{32}Y_{2,-1} - R_{31}Y_{1,-1})
\end{aligned}
\tag{29}
$$

which have opposite magnetic charge. Just as uniform magnetic field bends electric charges (into circles) the uniform electric field will bend the two oppositely magnetically charged states in opposite directions. A SQUID detector placed in the path of one of the two beams can detect the magnetic charge.

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